Sample Paper – 2
Mathematics Class X

Time: Three hours **Max. Marks: 80**

General Instructions:

1. All Questions are compulsory.

2. The question paper consists of thirty questions divided into 4 sections A, B, C and D. Section A comprises of ten questions of 01 mark each, section B comprises of five questions of 02 marks each, section C comprises of ten questions of 03 marks each and section D comprises of five questions of 06 marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.

5. In question on construction, drawings should be neat and exactly as per the given measurements.

6. Use of calculators is not permitted. However, you may ask for mathematical tables.

Section A (Each 01 mark)

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1. Write 98 as product of its prime factors.

2. The sum and product of the zeroes of a quadratic polynomial are - ½ and -3 respectively. What is the quadratic polynomial?

3. Find the value of p for which the equation $2x^2 - \sqrt{2}px + p = 0$ has real and equal roots.

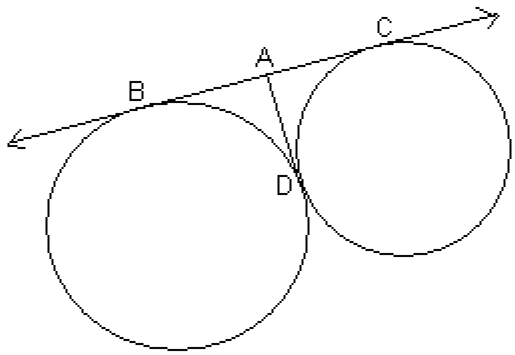
4. How many terms of A.P 20, 18, 16 … are needed to give the sum zero?

5. What is the angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree?

6. The surface area of a sphere is 616 cm^2; find the volume of the sphere.

7. Triangle ABC is such that AB = 3 cm, BC = 2 cm, and CA = 2.5 cm. If $\triangle DEF \sim \triangle ABC$ and EF = 4cm; find the perimeter of $\triangle DEF$.

8. In the adjoining figure, BC is a common tangent to the circles which touches externally at D. If BA = 4.2 cm, then find the length of BC.



9. Find the probability of not getting a sum 7, in a single throw of a pair of dice.

10. Find the Median of the first 10 multiples of five.

Section B (Each 02 marks)

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11. Find the measure of angles of a cyclic quadrilateral ABCD where $\angle A = (2x - 1)^\circ$, $\angle B = (y + 5)^\circ$, $\angle C = (2y + 15)^\circ$ and $\angle D = (4x - 7)^\circ$.

12. Evaluate: $\frac{2\sin 68^o}{\cos 22^o} - \frac{2\cot 15^o}{5\tan 75^o} - \frac{3\tan 45^o.\tan 20^o.\tan 40^o.\tan 50^o.\tan 70^o}{5}$

13. If the centroid of a triangle is (1, 4) and two of its vertices are (4, – 3) and (– 9, 7), then find the coordinates of third vertex of the triangle.

14. BL and CM are the medians of a triangle ABC right-angled at A. Prove that $4(BL^2 + CM^2) = 5\,BC^2$.

Or

In a ΔABC, $AD \perp BC$ and $AD^2 = BD \times CD$. Prove that ΔABC is a right triangle.

15. In a bag slips numbered 1 to 30 are mixed up together and then a slip is drawn at random. What is the probability that slip has a number which is multiple of 2 or 5?

Section C (Each 03 marks)

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16. Show that one and only one out of n, n + 2 or n + 4 is divisible by 3, where n is any positive integer.

Or

Using Euclid's division algorithm, find the HCF of 26565, 25806, and 20930.

17. Verify that 3, – 1 and $-\frac{1}{3}$ are the zeros of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$ and then verify the relationship between the zeros and its coefficients.

18. 10 students of class X took part in a General knowledge quiz. The number of girls is 4 more than the number of boys. Form the pair of linear equation in this problem and find the number of girls and boys by using graphical method.

19. Rekha joined the school in year 1998 on a monthly salary of Rs 6,500 and an annual increment of Rs 400 after 1998. In which year her monthly salary would be Rs 10,500.

20. Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ecA + \cot A$

21. Find the area of quadrilateral with vertices (1, 1), (7, – 3), (12, 2) and (7, 21).

Or

If the coordinates of two points A and B are (3, 4) and (5, – 2) respectively and PA = PB and area of ΔPAB = 10, then find the coordinates of point P.

22. In what ratio does the line x – y – 2 = 0 divide the line segment joining the coordinates (3, – 1) and (8, 9)?

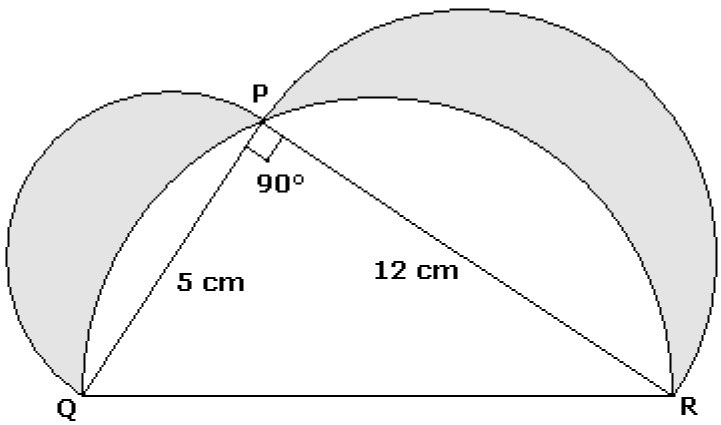
23. Draw a circle of radius 4.2 cm. Take a point at a distance of 7.8 cm from the centre of the circle, draw two tangents to the circle.

24. Two concentric circles are of radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the smaller circle.

25. A square is inscribed in a quadrant of a circle. If the side of the square is 25 cm, find the area of the remaining part of the quadrant of the circle.

Or

Triangle PQR is right angled triangle rights angled at P. Semicircles are drawn on PQ, QR and PR as diameters. Find the area of the shaded region as in adjoining figure.



Section D (Each 06 mark)

26. An express train covers a journey of 240 km at a constant speed. A passenger train has a speed of 12 km/hr lesser than the express train. It takes one hour more than the express train to cover the journey. Find the speed of express train.

Or

Nine times the side of one square exceeds the perimeter of a second square by one meter and six times the area of the second square exceeds 29 times the area of the first by one square meter .Find the side of each square.

27. A garden keeper sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of a river and in line with the foot of the tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively; find the width of the river.

Or

From a window 15 m high above the ground in the street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45° respectively, what is the height of the opposite house?

28. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Use the above theorem, in the following.
The areas of two similar triangles are 81 cm^2 and 144 cm^2. If the largest side of the smaller triangle is 27 cm, find the largest side of the larger triangle.

29. A bucket made up of a metal sheet is in the form of frustum of a cone of height 16 cm with diameters of its lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also find the cost of the bucket if the cost of metal sheet used is Rs 20 per 100 cm^2. (user $\pi = 3.14$).

30. Find the modal monthly expenditure:

Monthly expenditure (in Rs)	**Number of workers**
1000 – 2000	12
2000 – 3000	15
3000 – 4000	10
4000 – 5000	13
5000 – 6000	17
6000 – 7000	10
7000 – 8000	12
8000 – 9000	11
Total	**100**

End

Solution

Section A (Each 01 mark)

1. $98 = 2 \times 49 = 2 \times 7 \times 7 = 2 \times 7^2$
Prime factors of 98 are 2, 7 and 7.

2. Here, the sum and product of the zeroes of a quadratic polynomial are – ½ and – 3 respectively. Let p(x) be the quadratic polynomial.
$P(x) = x^2 - (\text{sum of roots})x + (\text{product of roots})$
$= x^2 - (-\frac{1}{2})x + (-3)$
$= x^2 + \frac{1}{2}x - 3$

= ½($2x^2$ + x − 6)
= k($2x^2$ + x − 6), where k (= ½) is a constant.

3. The given quadratic equation is $2x^2 - \sqrt{2}px + p = 0$.
On comparing with $ax^2 + bx + c = 0$, we get
a = 2, b = $-\sqrt{2}$ p and c = p
For equal and real roots, Determinant = 0
i.e. $b^2 - 4ac = 0$

$\Rightarrow \left(-\sqrt{2}p\right)^2 - 4\times 2p = 0$

$\Rightarrow 2p^2 - 8p = 0$

$\Rightarrow 2p(p-4) = 0$

$\Rightarrow p = 0, p = 4$

4. Here, a = 20, d = − 2, S_n = 0, n =?

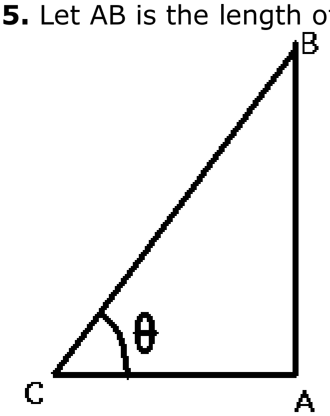
$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 0 = \frac{n}{2}[40 + (n-1)(-2)]$

$\Rightarrow 40 - 2n + 2 = 0$

$\Rightarrow n = 21$

5. Let AB is the length of the tree and AC be the length of the shadow.



$\angle ACB = \theta$ and $AC = \sqrt{3}AB$

Then $\cot\theta = \dfrac{AC}{AB}$

$\Rightarrow \cot\theta = \sqrt{3} = \cot 30^o$

$\Rightarrow \theta = 30^o$

6. Let r be the radius of the sphere.

$4\pi r^2 = 616$

$\Rightarrow r^2 = \dfrac{616 \times 7}{4 \times 22} = 49$

$\Rightarrow r = 7$

$\text{Volume} = \dfrac{4}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 7 = \dfrac{4312}{3} \text{cm}^3$

7. Perimeter of $\triangle ABC$ = 3 + 2 + 2.5 = 7.5 cm

$\triangle ABC \sim DEF \quad (\text{Given})$

$\therefore \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD} = \dfrac{AB + BC + CA}{DE + EF + FD} \quad \left[\begin{array}{l}\text{Ratio of correspondiong} \\ \text{side of similar } \triangle\text{s is same as} \\ \text{ratio of their perimeters}\end{array}\right]$

$\Rightarrow \dfrac{BC}{EF} = \dfrac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF}$

$\Rightarrow \dfrac{2}{4} = \dfrac{7.5}{\text{Perimeter of } \triangle DEF}$

$\Rightarrow \text{Perimeter of } \triangle DEF = \dfrac{4 \times 7.5}{2} = 15 \text{ cm}$

8. If two circles touch each other at a point externally, then the line from the point where the circles touch is bisect the common.

∴ AB = AC = ½ BC

⇒ BC = 2AB

⇒ BC = 2 × 4.2 = 8.4 cm

9. Total favorable cases n(S) = $(6)^2$ = 36

Cases of getting a sum of seven in a pair of dice = A $=\{(1,6),(2,5),(3,4),(4,3),(6,1),(5,2)\}$

$\therefore n(A) = 6$

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Probability of not getting the sum of 7 = $P\left(\overline{A}\right)$

$P\left(\overline{A}\right) = 1 - P\left(A\right) \Rightarrow P\left(\overline{A}\right) = 1 - \frac{1}{6} = \frac{5}{6}$

10. The observations are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
Here, n = 10, which is an even number

Therefore, the median is the average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

i.e., the median is the average of $\left(\frac{10}{2}\right)^{th}$ and $\left(\frac{10}{2}+1\right)^{th}$ observations

i.e., the median is the average of 5^{th} and 6^{th} observations.

Median = $\frac{25+30}{2} = \frac{55}{2}$ = 27.5

Section B (Each 02 marks)

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11. We know that the sum of the opposite angles of a cyclic quadrilateral is 180°.
In the cyclic quadrilateral ABCD, angles A and C and angles B and D form pairs of opposite angles.

$\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180$

$\Rightarrow \quad 2x - 1 + 2y + 15 = 180$ and $y + 5 + 4x - 7 = 180$

$\Rightarrow \quad 2x + 2y = 166$ and $4x + y = 182$

$\Rightarrow \quad 2x - 1 + 2y + 15 = 180$ and $y + 5 + 4x - 7 = 180$

$\Rightarrow \quad x + y = 83$ ...(i) and

$4x + y = 182$ ...(ii)

On subtracting equation (i) from equation (ii); we get

$3x = 99 \Rightarrow x = 33$

Substituting x = 33 in equation (i); we get y = 50
Hence,

$\angle A = (2 \times 33 - 1)^\circ = 65$

$\angle B = (50 + 5)^\circ = 55$

$\angle C = (2 \times 50 + 15)^\circ = 115$

$\angle D = (4 \times 33 - 7)^\circ = 125$

12.

$$\frac{2\sin 68^o}{\cos 22^o} - \frac{2\cot 15^o}{5\tan 75^o} - \frac{3\tan 45^o.\tan 20^o.\tan 40^o \tan 50.\tan 70^o}{5}$$

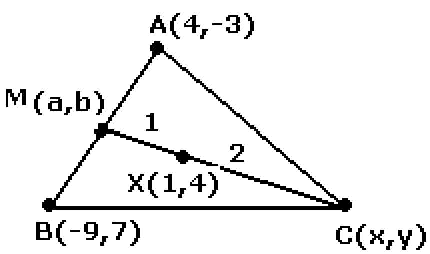
$$= \frac{2\sin\left(90^o - 22^o\right)}{\cos 22^o} - \frac{2\cot\left(90^o - 75^o\right)}{5\tan 75^o} - \frac{3\times 1.\tan\left(90^o - 70^o\right).\tan\left(90^o - 50\right)\tan 50^o \tan 70^o}{5}$$

$$= \frac{2\cos 22^o}{\cos 22^o} - \frac{2\tan 75^o}{5\tan 75^o} - \frac{3\cot 70^o.\cot 50^o.\tan 50^o.\tan 70}{5}$$

$$= 2 - \frac{2}{5} - \frac{3}{5} \qquad \left[\because \tan\theta \cot\theta = 1\right]$$

$$= 2 - \frac{5}{5} = 2 - 1 = 1$$

13.



Since, X is the centroid and CM is the median then X divides the median in the ratio 2 **:** 1.
Now, M is the mid-point of AB,

$$\therefore\ a = \frac{4 + (-9)}{2} \text{ and } b = \frac{-3 + 7}{2}$$

$$\Rightarrow\ a = \frac{-5}{2} \text{ and } b = \frac{4}{2} = 2$$

Applying section formula we find the coordinates of C

$$x = \frac{2\times\frac{-5}{2} + 1\times x}{2 + 1} = \frac{-5 + x}{3} = 1$$

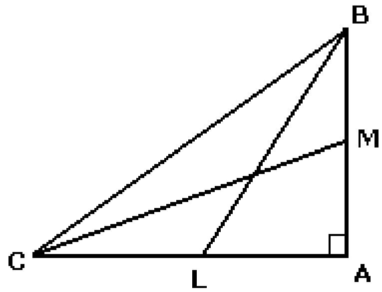
$$\Rightarrow\ -5 + x = 3 \ \Rightarrow\ x = 8$$

$$\text{Also, } \frac{4 + y}{3} = 4$$

$$\Rightarrow\ 4 + y = 12 \ \Rightarrow\ y = 8$$

Thus, the coordinates of C is (8, 8).

14. In ΔABC, BL and CM are the medians and $\angle A = 90^o$



In right triangle BAC,
$BC^2 = AB^2 + AC^2$ ...(1)
In right triangle BAL,

$$BL^2 = AB^2 + AL^2$$

$$\Rightarrow \quad BL^2 = AB^2 + \left(\frac{1}{2}AC\right)^2$$

$$\Rightarrow \quad BL^2 = AB^2 + \frac{1}{4}AC^2$$

$$\Rightarrow \quad 4BL^2 = AC^2 + 4AB^2 \qquad ...(ii)$$

In right triangle MAC,

$$CM^2 = AC^2 + AM^2$$

$$\Rightarrow \quad CM^2 = AC^2 + \left(\frac{1}{2}AB\right)^2$$

$$\Rightarrow \quad CM^2 = AC^2 + \frac{1}{4}AB^2$$

$$\Rightarrow \quad 4CM^2 = AB^2 + 4AC^2 \qquad ...(iii)$$

On adding equations (ii) and (iii), we get

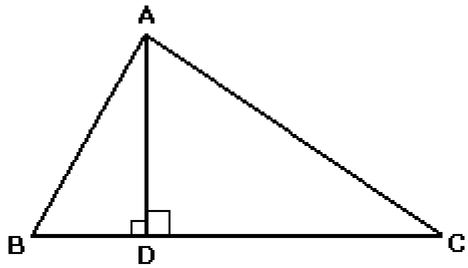
$$4BL^2 + 4CM^2 = AC^2 + 4AB^2 + AB^2 + 4AC^2$$

$$\Rightarrow \quad 4\left(BL^2 + CM^2\right) = 5\left(AC^2 + AB^2\right)$$

$$\Rightarrow \quad 4\left(BL^2 + CM^2\right) = 5\left(BC^2\right) \qquad [By\, eq^n\ (i)]$$

Or

In a $\Delta ABC,\ AD \perp BC$



In right triangles ADB and ADC, we have

$AB^2 = AD^2 + DB^2$ ...(i) and

$AC^2 = AD^2 + DC^2$ ...(ii)

On adding equations (i) and (ii), we get

$AB^2 + AC^2 = AD^2 + DB^2 + AD^2 + DC^2$

$= 2\,AD^2 + DB^2 + DC^2$

$= 2(BD \times CD) + DB^2 + DC^2$ [Given that $AD^2 = BD \times CD$]

$= BD^2 + CD^2 + 2(BD \times CD)$

$= (BD + CD)^2$

$= BC^2$

Thus, we have $AB^2 + AC^2 = BC^2$

So, by converse of Pythagoras theorem ΔABC is a right triangle, right angled at A.

15. Total slips = 30

$n(S) = 30$

Let A event be slip drawn has a number multiple of 2 or 5.

A = Multiple of 2 + multiple of 5 – common multiple of 2 and 5 =

= {2, 4, 6, ..., 30} + {5, 10, 15, ..., 30} – {10, 20, 30}

n(A) = 15 + 6 – 3 18

$\therefore$ Required Probability $P(A) = \frac{n(A)}{n(S)} = \frac{18}{30} = \frac{3}{5}$

Section C (Each 03 marks)

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16. Any positive integer n is of the form 3q or 3q + 1 or 3q + 2 for some integer q

Case (i) when n = 3q, which is divisible by 3

$\Rightarrow$ n + 2 = 3q + 2,

$\Rightarrow$ n + 2 leaves remainder 2 when divided by 3

$\Rightarrow$ n + 2 is not divisible by 3

Again n=3q,

$\Rightarrow$ n + 4 = 3q + 4 = 3(q + 1) + 1,

⇒ $n + 4$ leaves remainder 1 when divided by 3
⇒ $n + 4$ is not divisible by 3

Case (ii) when $n = 3q + 1$,
⇒ n leaves remainder 1 when divided by 3
⇒ n is not divisible by 3

Again $n = 3q + 1$,
⇒ $n + 2 = (3q + 1) + 2 = 3(q + 1)$,
⇒ $n + 2$ is divisible by 3

Again $n = 3q + 1$,
⇒ $n + 4 = 3q + 1 + 4 = 3(q + 1) + 2$,
⇒ $n + 4$ leaves remainder 2 when divided by 3
⇒ $n + 4$ is not divisible by 3
Thus, $(n + 2)$ is divisible by 3. But n and $(n + 4)$ are not divisible by 3

Similarly, we can show that for $n = 3q + 2$, $n + 4$ is divisible by 3, But n and $n + 2$ are not divisible by 3.

Or

Step 1: 26565 > 25806, applying lemma to 26565 and 25806, we get
$26565 = 25806 \times 1 + 759$
Step 2: Remainder $759 \neq 0$, we apply division lemma to 25806 and 759, we get
$25806 = 759 \times 34 + 0$
Step 3: Remainder is zero, so HCF of 26565 and 25806 is 759. So now find HCF of 759 and 20930, Now 20930>759, applying division lemma to 20930 and 759, we get
$20930 = 759 \times 27 + 437$
Step 4: Remainder $437 \neq 0$, we apply division lemma to 759 and 437, we get
$759 = 437 \times 1 + 322$
Step 5: Remainder $322 \neq 0$, we apply division lemma to 437 and 322, we get
$437 = 322 \times 1 + 115$
Step 6: Remainder $115 \neq 0$, we apply division lemma to 322 and115, we get
$322 = 115 \times 2 + 92$
Step 7: Remainder $92 \neq 0$, we apply division lemma to 115 and 92, we get
$115 = 92 \times 1 + 23$
Step 8: Remainder $23 \neq 0$, we apply division lemma to 92 and 23, we get
$92 = 23 \times 4 + 0$
Step 9: Since remainder is 0 and divisor at this stage is 23, HCF of 26565, 25806 and 20930 is 23.

17. We have

$p(x) = 3x^3 - 5x^2 - 11x - 3$

Now, $p(3) = 3(3)^3 - 5(3)^2 - 11(3) - 3$

$= 81 - 45 - 33 - 3 = 0$,

$p(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3$

$= -3 - 5 + 11 - 3 = 0$ and

$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^3 - 5\left(-\frac{1}{3}\right)^2 - 11\left(-\frac{1}{3}\right) - 3$

$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3 = 0$

So, 3, – 1 and $-\frac{1}{3}$ are the zeros of polynomial $p(x)$

Let $\alpha = 3, \beta = -1$ and $\gamma = -\frac{1}{3}$. Then

$$\alpha + \beta + \gamma = 3 - 1 - \frac{1}{3} = \frac{5}{3} = -\left(-\frac{5}{3}\right) = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times (-1) + (-1)\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) \times 3$$

$$\Rightarrow \quad \alpha\beta + \beta\gamma + \gamma\alpha = -3 + \frac{1}{3} - 1 = \frac{-11}{3} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = 3 \times (-1) \times \left(-\frac{1}{3}\right) = 1 = -\left(\frac{-3}{3}\right) = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

18. Let x and y be the number of girls and boys respectively who took part in the quiz. Then

x + y = 10 …(1) and

x – y = 4 …(2)

By equation (1), we have

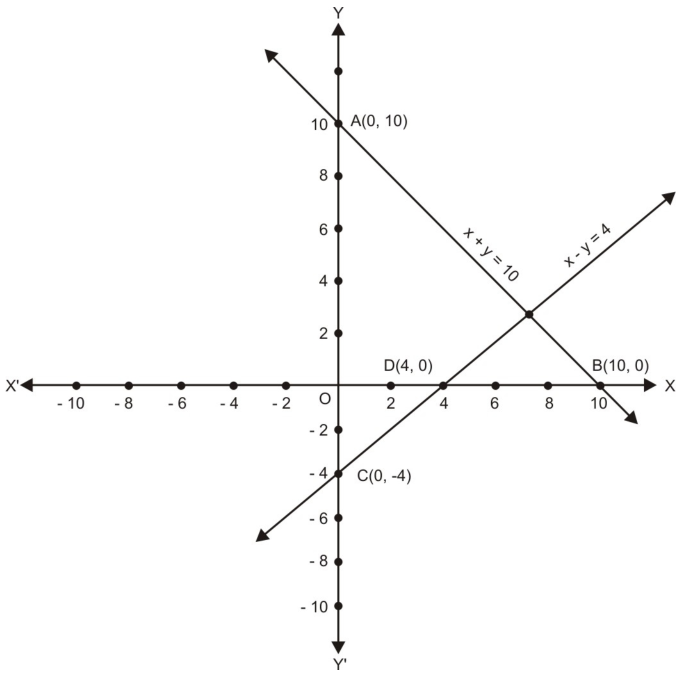
y = 10 – x

x	0	4	7	10
y	10	6	3	0
	A	**B**	**C**	**D**

By equation (2), we have

y = x – 4

x	0	4	7	10
y	−4	0	3	6
	E	**F**	**G**	**H**



Thus, lines x + y = 10 and x − y = 4 intersect at the point (7, 3). So, x = 7, y = 3 is the required solution, i.e. the number of girls is 7 and number of boys is 3.

19. Let the salary be Rs 10500 in the n^{th} year.

Her annual salary form an A.P. with the first term Rs 6500 and common difference is Rs 500.

Monthly salary of Rekha in 1998 = Rs 6500

a = 6500

Increment in annual income every year = Rs 400

∴ d = 400

Paid salary = 10500

i.e. $a_n = 10500$

⇒ $a_n = a + (n - 1)d$

10500 = 6500 + (n − 1)400

⇒ 4000 = 400n − 400

⇒ 4400 = 400n

⇒ n = 11

∴ After 11 year i.e. in 2008 her monthly salary will be Rs 10500.

20.

$$\text{LHS} = \frac{(\cos A - \sin A) + 1}{(\cos A + \sin A) - 1}$$

$$= \frac{(\cos A - \sin A) + 1}{(\cos A + \sin A) - 1} \times \frac{(\cos A + \sin A) + 1}{(\cos A + \sin A) + 1}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A) + \cos A - \sin A + \cos A + \sin A + 1}{(\cos A + \sin A)^2 - (1)^2}$$

$$= \frac{\cos^2 A - \sin^2 A + 2\cos A + 1}{\cos^2 A + \sin^2 A + 2\cos A \sin A - 1}$$

$$= \frac{\cos^2 A - \sin^2 A + 2\cos A + 1}{2\cos A \sin A}$$

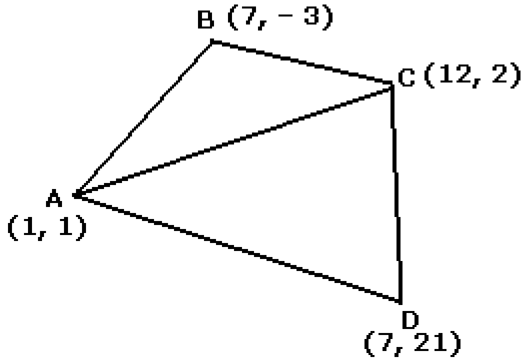
$$= \frac{\cos^2 A - 1 + \cos^2 A + 2\cos A + 1}{2\cos A \sin A}$$

$$= \frac{2\cos A(1 + \cos A)}{2\cos A \sin A}$$

$$= \frac{1 + \cos A}{\sin A} = \frac{1}{\sin A} + \frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A + \cot A = \text{RHS}$$

21. Area of quad = ar (ABC) + ar(ACD)



Area of $\Delta ABC = \frac{1}{2}\{1(-3-2)+7(2-1)+12(1+3)\}$

$= \frac{1}{2}\{1(-5)+7(1)+12(4)\}$

$= \frac{1}{2}\{-5+7+48\}$

$= \frac{1}{2}\{50\} = |25| = 25$ sq.units

Area of $\Delta ACD = \frac{1}{2}\{1(2-21)+12(21-1)+7(1-2)\}$

$= \frac{1}{2}\{1(-19)+12(20)+7(-1)\}$

$= \frac{1}{2}\{-19+240-7\}$

$= \frac{1}{2}\{-26+240-7\} = \frac{1}{2} \times 214 = 107$ sq.units

$\therefore$ Area of quadrilateral $= ar(\Delta ABC) + ar(\Delta ACD)$

$= 25 + 107 = 132$ sq.units

Or

Let the coordinates of P be (x, y). Then PA = PB
Or $PA^2 = PB^2$
$\Rightarrow$ $(x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$
$\Rightarrow$ $x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2 + 4y + 4$
$\Rightarrow$ $4x - 4 - 12y = 0$

⇒ x – 3y –1 = 0 ...(1)

Area of $\Delta PAB = 10$

Now, the coordinates are P(x, y), A(3, 4) and (5, – 2). Thus

$$\text{Area of } \Delta PAB = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$i.e.\ 10 = \frac{1}{2}\{x(4+2) + 3(-2-y) + 5(y-4)\}$$

$$\Rightarrow\ 10 = \frac{1}{2}\{6x - 6 - 3y + 5y - 20\}$$

$$\Rightarrow\ 10 = \frac{1}{2}\{6x + 2y - 26\}$$

$$\Rightarrow\ 20 = \{6x + 2y - 26\}$$

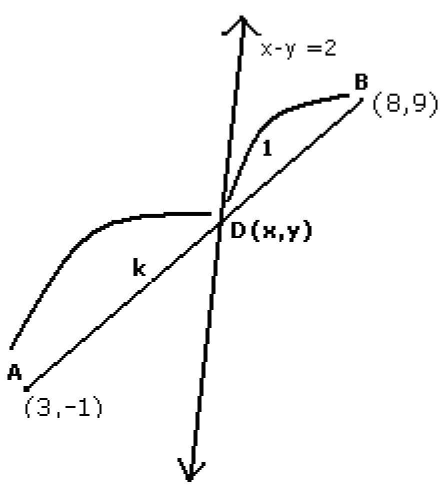
$$\Rightarrow\ 46 = 6x + 2y$$

$$\Rightarrow\ 3x + y - 23 = 0 \qquad ...(2)$$

Solving (1) and (2), we get

$x = 7$ and $y = 2$

22. Suppose the line x – y – 2 = 0 divides the line joining the points (3, – 1) and (8, 9) in the ratio of k **:** 1. Then



$x = \dfrac{8k+3}{k+1}$ ...(1) and $y = \dfrac{9k-1}{k+1}$ ...(2)

Since, the equation of the line is: $x - y = 2$

$\therefore \dfrac{8k+3}{k+1} - \dfrac{9k-1}{k+1} = 2$

$\Rightarrow \dfrac{8k+3-9k+1}{k+1} = 2$

$\Rightarrow 8k+3-9k+1 = 2(k+1)$

$\Rightarrow 8k+3-9k+1 = 2k+2$

$\Rightarrow 8k-9k-2k = 2-3-1$

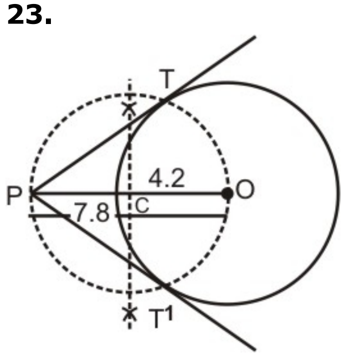
$\Rightarrow -3k = -2$

$\Rightarrow k = \dfrac{2}{3}$

$\Rightarrow k:1 = \dfrac{2}{3}:1 = 2:3$

Thus, the line x − y = 2 divides the line joining the points (3, − 1) and (8, 9) in the ratio of 2 **:** 3 internally.

23.

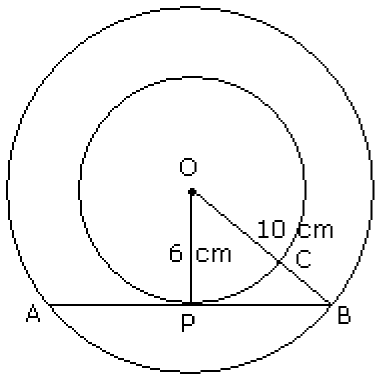


(Note: Don't write steps in examination; these are here only for understanding)
Steps of Construction:

(1) Take a point O on the plane and draw a circle of given radius 4.2 cm.
(2) Take a point at a distance of 7.8 cm from the centre O of the circle and join OP.
(3) Draw perpendicular bisector of OP, intersecting OP at C.
(4) Taking C as centre and OC = PC as radius, draw a circle to intersect the given circle at T and T'
(5) Join PT and PT'

Thus, PT and PT' are the required tangents to the circle.

24. Let O be the common centre of two concentric circles whose radii are OA (= 10 cm) and OC (= 6 cm) respectively.



Let AB be the chord of larger circle touching the smaller circle at P. Join OP. So, AB is tangent to the smaller circle at P.

$\therefore\ OP \perp AB$

Also, we know that perpendicular drawn from the centre to any chord of circle, bisects the chord.

So, AP = PB

Now, in right triangle OPB,

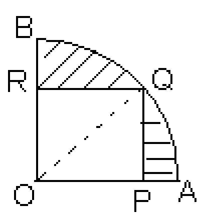
$PB^2 = OB^2 - OP^2$

$\Rightarrow\ PB = \sqrt{(10)^2 - (6)^2}$

$= \sqrt{100 - 36} = \sqrt{64} = 8$ cm

Length of the chord of larger circle = AB = 2(PB) = 2(8) = 16 cm

25. Area of the square OPQR = 25 × 25 = 625 cm^2



As OPQ is a right angled triangle

$\therefore\ OQ^2 = OP^2 + PQ^2$

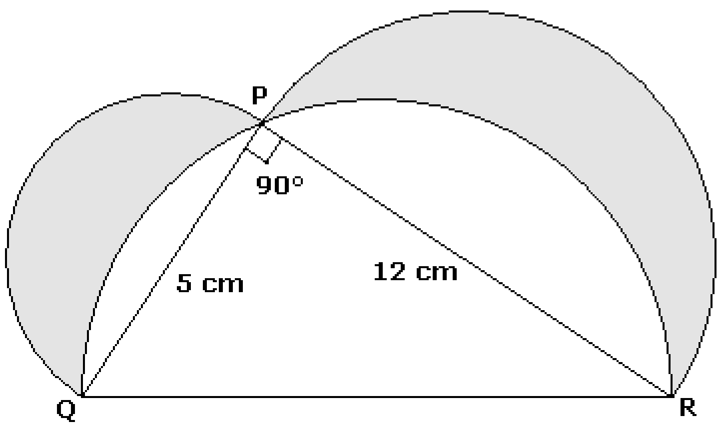
$= (25)^2 + (25)^2 = 1250\ cm^2$

Area of quadrant OAQB = $\frac{1}{4} \times \pi \times (OQ)^2\ cm^2$

$$= \frac{1}{4} \times 3.14 \times 1250 = 981.25 \text{ cm}^2$$

Area of the remaining part of the quadrant of circle = Area of the quadrant – Area of the square = 981.25 – 625 = 356.25 cm^2

Or



In right angled ΔPQR,

$$QR = \sqrt{PQ^2 + PR^2} = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13 \text{ cm}$$

Area of semi circle on side PQ = $\frac{1}{2} \times \frac{22}{7} \times \left(\frac{5}{2}\right)^2 \text{ cm}^2$

Area of semi circle on side PR = $\frac{1}{2} \times \frac{22}{7} \times \left(\frac{12}{2}\right)^2 \text{ cm}^2$

Area of semi circle on side QR = $\frac{1}{2} \times \frac{22}{7} \times \left(\frac{13}{2}\right)^2 \text{ cm}^2$

Area of triangle PQR = ½ × 5 × 12 = 30 cm^2

Area of the shaded region = Area of semi circle on side PQ + Area of semi circle on side PR + Area of triangle PQR – Area of semi circle on side QR

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{5}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{12}{2}\right)^2 + 30 - \frac{1}{2} \times \frac{22}{7} \times \left(\frac{13}{2}\right)^2$$

$$= 30 + \frac{1}{2} \times \frac{22}{7}\left[\left(\frac{5}{2}\right)^2 + \left(\frac{12}{2}\right)^2 - \left(\frac{13}{2}\right)^2\right]$$

$$= 30 + \frac{11}{7}\left[\frac{25 + 144 - 169}{4}\right]$$

$$= 30 + \frac{11}{7} \times \frac{0}{4}$$

$$= 30 + 0$$

$$= 30 \text{ cm}^2$$

Section D (Each 06 mark)

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26. Let the speed express train be x km/hr. The speed of the passenger train = (x – 12) km/hr.

∴ Time taken by express train and passenger train to cover 240 km is $\frac{240}{x - 12}$ and $\frac{240}{x}$ hrs respectively.

Since passenger train takes one hour more than the express train

$$\therefore \frac{240}{x - 12} = \frac{240}{x} + 1$$

$$\Rightarrow 240x = 240x - 12 \times 240 + x^2 - 12x$$

⇒ $x^2 - 12x - 2880 = 0$

⇒ $x^2 + 48x - 60x - 2880 = 0$

⇒ $(x - 60)(x + 48) = 0$

⇒ $x = 60$ or $x = -48$

But – 48 is not acceptable.

∴ Speed of express train = 60 km/hr

Or

Let the sides of the squares be x and y meters. Their perimeters are 4x & 4y and areas are x^2 & y^2 respectively.

Nine times the side of one square exceeds the perimeter of a second square by one meter;

$9x - 4y = 1$ …(1)

According to second condition,

$6y^2 - 29x^2 = 1$

$\Rightarrow \quad 6y^2 - 29\left(\frac{1+4y}{9}\right)^2 = 1 \qquad [By\ eq^n\ (1)]$

$\Rightarrow \quad 6y^2 - \frac{29}{81}(1+16y^2+8y) = 1$

$\Rightarrow \quad 486y^2 - 29 - 464y^2 - 232y = 81$

$\Rightarrow \quad 22y^2 - 232y - 110 = 0$

$\Rightarrow \quad 11y^2 - 116y - 55 = 0$

$D = (116)^2 + 4 \times 11 \times 55$

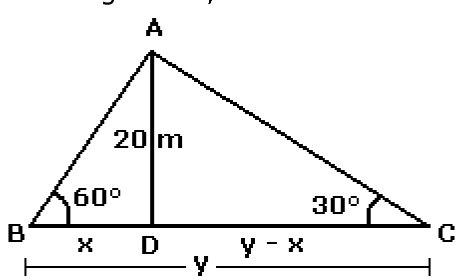
$\quad = 13456 + 2420 = 15876$

$y = \frac{116 \pm \sqrt{15876}}{22} = \frac{116 \pm 126}{22}$

$\Rightarrow \quad y = 11$ and $y = \frac{-5}{11}$ (not acceptable)

Put y = 11 in (i);
9(x) – 4(11) =1
$\Rightarrow$ 9x – 44 = 1
$\Rightarrow$ x = 45/9 = 5
Hence, the sides of the square are 5 units and 11 units respectively.

27. Let AD be the 20 m high tree; AB = 20 m and B and C are the banks of river.
In triangle ADB,



$$\frac{AD}{BD} = \tan 60^o = \sqrt{3}$$

$$\Rightarrow \frac{20}{x} = \sqrt{3} \Rightarrow x = \frac{20}{\sqrt{3}}$$

In ΔADC,

$$\frac{AD}{DC} = \tan 30^o = \frac{1}{\sqrt{3}}$$

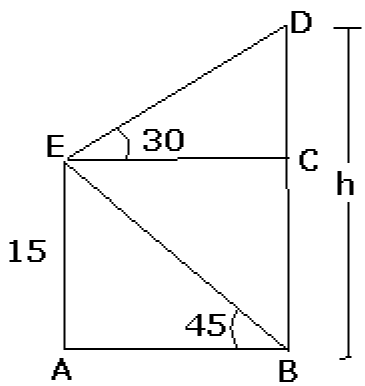
$$\Rightarrow \frac{20}{y - x} = \frac{1}{\sqrt{3}} \Rightarrow y - x = 20\sqrt{3}$$

$$\Rightarrow y - \frac{20}{\sqrt{3}} = 20\sqrt{3}$$

$$\Rightarrow y = 20\sqrt{3} + \frac{20}{\sqrt{3}} = \frac{60 + 20}{\sqrt{3}} = \frac{80}{\sqrt{3}} \text{ m}$$

Or

Let the window be at a height E which is 15 m above the ground, BD be the house on the opposite side of the street and we have to find the value of BD.



In ΔEAB,

$$\frac{EA}{AB} = \tan 45^o = 1 \Rightarrow \frac{15}{AB} = 1 \Rightarrow AB = 15$$

In ΔECD,

$$\frac{DC}{EC} = \tan 30^o = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h-15}{15} = \frac{1}{\sqrt{3}} \quad [\because EC = AB]$$

$$\Rightarrow h - 15 = \frac{15}{\sqrt{3}}$$

$$\Rightarrow h - 15 = 5\sqrt{3}$$

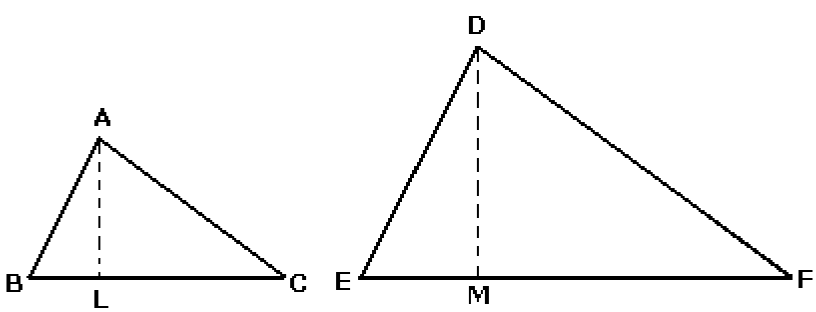
$$\Rightarrow h = 15 + 5\sqrt{3}$$

28. First Part:

Given: Two triangles ABC and DEF, such that $\Delta ABC \sim \Delta DEF$.

To Prove: $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction: Draw $AL \perp BC$ and $DM \perp EF$



Proof: Since, similar triangles are equiangular and their sides are proportional. Therefore,

$\Delta ABC \sim \Delta DEF \Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ...(1)

In ΔALB and ΔDME,

$\angle B = \angle E$ [From (1)] and

$\angle ALB = \angle DME = 90^o$

So, by A-A criterion of similarity, we have

$\Delta ALB \sim \Delta DME$

$$\Rightarrow \frac{AB}{DE} = \frac{AL}{DM} \quad ...(2)$$

From (1) and (2), we get

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM} \quad ...(3)$$

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF} \quad [\text{By } eq^n (3)]$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC^2}{EF^2}$$

Also, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Hence, $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

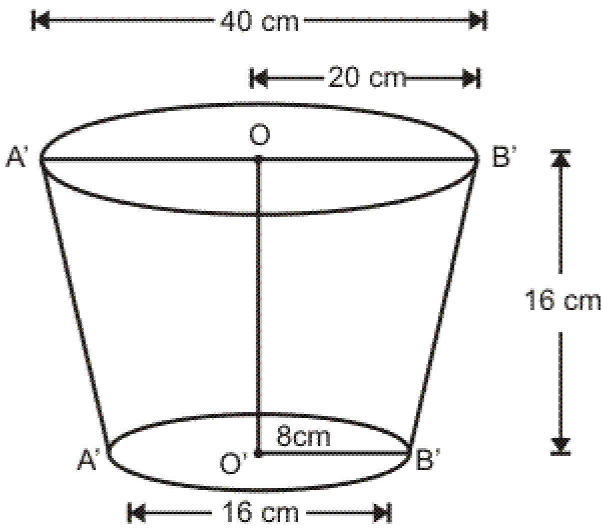
Second Part: Let x be the larger side of larger triangle.
Area of larger triangle = 144 cm^2 and
Area of smaller triangle = 81 cm^2
From the above result,

$$\frac{144}{81} = \frac{x^2}{(27)^2} \Rightarrow x^2 = \frac{144 \times 27 \times 27}{81} = 144 \times 9$$

$$\Rightarrow x = 12 \times 3 = 36 \text{ cm}$$

Hence, the length of larger side of larger triangle is 36 cm.

29. Let h be the height and R and r be the radii of upper and lower ends of a frustum of a cone, i.e., bucket, respectively.



Then, h = 16 cm, R= $\frac{40}{20}$ = 20 cm and r = $\frac{16}{2}$ = 8 cm

Volume of the frustum of a cone, i.e., a bucket

$= \frac{\pi h}{3}\left[R^2 + r^2 + Rr\right]$

$= \frac{3.14 \times 16}{3}\left[(20)^2 + (8)^2 + 20 \times 8\right] \text{cm}^3$

$= \frac{3.14 \times 16}{3}\left[400 + 64 + 160\right] \text{cm}^3$

$= \frac{3.14 \times 16}{3} \times [624] \text{cm}^3$

$= 3.14 \times 16 \times 208 \text{ cm}^3$

$= 10449.92 \text{ cm}^3$

Now, l = Slant height of frustum of a cone, i.e., bucket

$=\sqrt{h^2 + (R - r)^2}$

$= \sqrt{(16)^2 + (20 - 80)^2}$

$= \sqrt{256+144}$ cm

$= \sqrt{400} = 20$ cm

Total surface area of the frustum of a cone, i.e., bucket

$= \pi\, l(\mathrm{R}+\mathrm{r}) + \pi\, r^2$

$= [3.14 \times 20 \times (20 + 8) + 3.14 \times (8)^2]\ \mathrm{cm}^2$

$= [3.14 \times 20 \times 28 + 3.14 \times 64]\ \mathrm{cm}^2$

$= 3.14(20 \times 28 + 64)\ \mathrm{cm}^2$

$= 3.14(560 + 64)\ \mathrm{cm}^2$

$= 1959.36\ \mathrm{cm}^2$

Cost of metal used to make the bucket in the form of the frustum of a cone

$= \mathrm{Rs}\left(\frac{20 \times 1959.36}{100}\right)$

$= \mathrm{Rs.}\ 391.87$

30. **Calculation of mode**

Monthly expenditure (in Rs)	Number of workers
1000 – 2000	12
2000 – 3000	15
3000 – 4000	10
4000 – 5000	13
5000 – 6000	17
6000 – 7000	10
7000 – 8000	12
8000 – 9000	11
Total	**100**

Since the frequency of the class 5000 – 6000 is greater, i.e., 17.
Therefore the modal class is 5000 – 6000.
ℓ = lower limit of the modal class = 5000
f_1 = frequency of the modal class = 17

f_0 = frequency of the class preceding the modal class = 13
f_2 = frequency of the class succeeding the modal class = 10
h = height (class size) of the modal class = 1000

$\therefore$ Mode = $\ell + \left(\frac{f_1 - f_o}{2f_1 - f_0 - f_2}\right) \times h$

$= 5000 + \left(\frac{17 - 13}{2 \times 17 - 13 - 10}\right) \times 1000$

= 5000 + 363.64 = 5363.64 (Approx.)

End
